Physics 513, Quantum Field Theory

Homework 10

Due Tuesday, 25th November 2003

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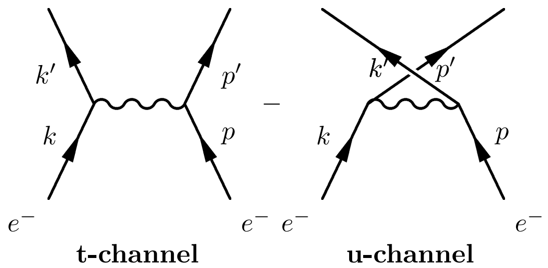
Electron-Electron Scattering We are to consider the elastic scattering of two electrons (Møller scattering) in Quantum Electrodynamics.

a) We are to draw the two tree-level Feynman diagrams for the scattering amplitude. We see that they are,

$$i\mathcal{M}_{\lambda'\mu';\lambda\mu} =$$



$$= ie^2\left[\bar{u}_{\mu'}(k')\gamma^\mu u_\mu(k)\frac{1}{(k-k')^2}\bar{u}_{\lambda'}(p')\gamma_\mu u_\lambda(p) - \bar{u}_{\mu'}(k')\gamma^\mu u_\lambda(p)\frac{1}{(p-k')^2}\bar{u}_{\lambda'}(p')\gamma_\mu u_\mu(k)\right].$$

The relative minus sign is a simple consequence of Fermi statistics.

b) Using the Gordon identity, derived in homework 5 problem 3, we are to derive a simple form of the amplitude for the forward most direction. Here we will assume that $p' \sim p$. So,

$$\begin{aligned}
\bar{u}_{\lambda'}(p')\gamma^\mu u_{\lambda'}(p) &= \bar{u}_{\lambda'}(p')\left[\frac{(p'+p)^\mu}{2m} + \frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m}\right]u_\lambda(p),\\
&= \bar{u}_{\lambda'}(p)\left[\frac{(p+p)^\mu}{2m} + \frac{i\sigma^{\mu\nu}(p-p)_\nu}{2m}\right]u_\lambda(p),\\
&= \bar{u}_{\lambda'}(p)\frac{p^\mu}{m}u_\lambda(p),\\
&= 2p^\mu\delta_{\lambda'\lambda}.
\end{aligned}$$

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c) In the forward most direction, it is clear that the denominator for the t-channel contribution is small and the denominator for the u-channel contribution is large so only the t-channel contributions are relevant. In the t-channel amplitude, it is clear that spin cannot flip so that spin of the initial and final particles are the same. Therefore, the important terms are $\mathcal{M}_{LL;LL}, \mathcal{M}_{RR;RR}, \mathcal{M}_{RL;RL}, \mathcal{M}_{LR;LR}$.

d) In contrast to part (c), only the u-channel contributions are important so the final spin states may be switched. So the important amplitudes are $\mathcal{M}_{LL;LL}, \mathcal{M}_{RR;RR}, \mathcal{M}_{LR;RL}, \mathcal{M}_{RL;LR}$.

e) Using parts (a) and (b), we may compute,

$$\begin{aligned}
\mathcal{M}_{PR;LR} &= e^2 \bar{u}_{\mu'}(k)\gamma^\mu u_\mu(k)\frac{1}{(k-k')^2}\bar{u}_{\lambda'}(p)\gamma_\mu u_\lambda(p),\\
&= \frac{e^2}{-2\vec{k}^2(1-\cos\theta)}4k^\mu p_\mu,\\
&= \frac{e^2}{-\vec{k}^2\sin^2\theta/2}k^\mu p_\mu,\\
&= \frac{e^2}{-\vec{k}^2\sin^2\theta/2}\left(\frac{E_{cm}^2}{4}+\vec{k}^2\right),\\
&= \frac{e^2\left(\frac{E_{cm}^2}{2}-m^2\right)}{-\left(\frac{E_{cm}^2}{4}-m^2\right)\sin^2\theta/2},\\
&= -\frac{2e^2\left(E_{cm}^2-2m^2\right)}{\left(E_{cm}^2-4m^2\right)\sin^2\theta/2}.
\end{aligned}$$

f) We now should compute the differential cross section with respect to the scattering angle θ.

$$\begin{aligned}
\frac{d\sigma}{d\cos\theta} &= 2\pi\frac{d\sigma}{d\Omega},\\
&= \frac{2\pi|\mathcal{M}|^2}{64\pi^2E_{cm}^2},\\
&= \frac{2\pi\alpha^2 4\left(E_{cm}^2-2m^2\right)^2}{4E_{cm}^2\left(E_{cm}^2-4m^2\right)^2\sin^4\theta/2},\\
\therefore \frac{d\sigma}{d\cos\theta} &= \frac{2\pi\alpha^2\left(E_{cm}^2-2m^2\right)^2}{E_{cm}^2\left(E_{cm}^2-4m^2\right)^2\sin^4\theta/2}.
\end{aligned}$$

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A Delicate Balance Consider the reactions $a+b\to a'+b'$ and $a'+b'\to a+b$. These four particles may all have different masses and different spins given by $s_a, s_b, s_{a'}, s_{b'}$. We are to compute the ratio of differential cross sections with respect to solid angle Ω for the two processes.

Because of the enormous symmetry of the two processes, it will suffice to demonstrate a calculation of one of the processes. We will assume the process is time reversal so that the amplitude squared is the same for both. Let us compute the differential cross section.

$$\begin{aligned}
\frac{d\sigma}{d\Omega}(a+b\to a'+b') &= \frac{\vec{p}}{2E_a2E_b|v_a-v_b|(2\pi)^2 4E_{cm}}\frac{|\mathcal{M}(\text{sum})|^2}{(2s_a+1)(2s_b+1)},\\
&= \frac{\vec{p}|\mathcal{M}(\text{sum})|^2}{64\pi^2E_aE_b\vec{p}(1/E_a+1/E_b)E_{cm}(2s_a+1)(2s_b+1)},\\
&= \frac{|\mathcal{M}(\text{sum})|^2}{64\pi^2E_aE_b(1/E_a+1/E_b)(E_a+E_b)(2s_a+1)(2s_b+1)},\\
&= \frac{|\mathcal{M}(\text{sum})|^2}{64\pi^2(E_a+E_b)^2(2s_a+1)(2s_b+1)},\\
&= \frac{|\mathcal{M}(\text{sum})|^2}{64\pi^2k^2(2s_a+1)(2s_b+1)}.
\end{aligned}$$

Now, it is clear by symmetry that this implies

$$\frac{d\sigma}{d\Omega}(a'+b'\to a+b) = \frac{|\mathcal{M}(\text{sum})|^2}{64\pi^2k'^2(2s_{a'}+1)(2s_{b'}+1)}.$$

$$\therefore \frac{\frac{d\sigma}{d\Omega}(a+b\to a'+b')}{\frac{d\sigma}{d\Omega}(a'+b'\to a+b)} = \frac{k'^2(2s_{a'}+1)(2s_{b'}+1)}{k^2(2s_a+1)(2s_b+1)}.$$

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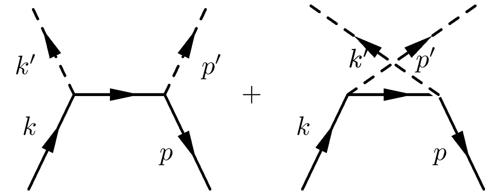
Fermion Annihilation in Yukawa Theory We are to consider the process of fermion anti-fermion annihilation into to scalars $f\bar{f} \to \phi\phi$.

a,b) The two Feynman diagrams for the S-matrix in the tree approximation are,

$i\mathcal{M} =$



t-channel **u-channel**

$$= (-ig^2)\left[u^2(p)\frac{\not{p} - \not{p}' + m}{(p-p')^2 - m^2}\bar{v}^r(k) + u^s(p)\frac{\not{p} - \not{k}' + m}{(p-k')^2 - m^2}\bar{v}^r(k)\right].$$

c) The relative sign is because of Bose statistics.